Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

MATHEMATICS IN THE SECONDARY SCHOOL, FROM THE POINT OF VIEW OF THE INDUSTRIAL ESTABLISHMENT.

By Sanford A. Moss.

We assume that we have only the case of a young man who is to enter industrial life upon leaving the secondary school and who is not to have a college education. As an introduction it is desirable to enumerate the various directions in which mathematical training received in the secondary school is of benefit to the young man in later industrial life.

The direction of benefit which seems to me by far the most important of all is "vocational directing"; the knowledge that the young man gains regarding his ability to do mathematical work. If he has enjoyed his mathematical courses, which usually means that he has secured good marks, or if the contrary is true, he is given a most important guide in selecting his life's vocation and in planning details of his career. Usually the young man is unconscious of the fact that he is being guided in this way. He has liked mathematical work in high school and unconsciously inclines towards a career which involves mathematics in some way, or he has disliked mathematical work and unconsciously inclines to a career which does not involve it.

"Vocational training" as a means of finding out what a young man is fitted for and starting him straight on his career is now being given considerable well-merited attention. It is usually supposed that this is a matter of trades and manual training. However, secondary school mathematics always has been, perhaps in an unrecognized way, just as much of an aid in vocation selection as a printing school or a manual training wood shop. When this point is fully recognized and when the general subject of vocation selection is as firmly established as it soon bids fair to be, mathematical work will be arranged so as to better secure benefit in this direction.

The direction of benefit which it seems to me should be placed second is the "mind training" in clear thinking given by a successful course in mathematics. One who can do clear thinking in a mathematical direction can usually do clear thinking in other directions. The reverse does not necessarily hold however. The training in clear thinking which is gained is of use in all other departments of life. This point has often been emphasized and need not be dwelt upon here.

The next direction of benefit I would make "Culture." A mathematical course properly conducted broadens one's outlook and opens up new realms whose existence could not otherwise be imagined. This point also has often been dwelt upon and need not be elaborated here.

It will probably be considered surprising that I place as a direction of benefit less important than the preceding "detailed knowledge of mathematical processes" for use in doing actual work in industrial life. Of course, this is an important direction in which benefit is received but I believe is subordinate to those before mentioned. The young man must have a general knowledge of the fundamental principles of geometry, trigonometry and algebra. However, he will rarely if ever in his later life encounter a problem at all like one which he learned in his text-book. Efforts are usually made to give so-called "practical problems" and there is no doubt that this is wise but no matter how practical we may try to be we can never expect to do more than to inculcate general principles, and leave the details to be worked out as they are met. A high school man in a mechanical drafting-room or a civil engineer's office makes little or no use of that part of his algebra where he was taught long division, least common multiple or such other specific subjects. It rarely happens even that he will be called upon to solve a simple equation, much less a quadratic equation. A young surveyor has little use for methods of solving triangles which he learned in high school. Most of the mathematical processes in industrial life are listed or tabulated or systematized or taken from an engineer's pocket-book in such a way that a young man who has to perform them has merely to follow out set forms which are supplied to him at the time, and has little or nothing to do in the way of exercising his originality. In order to intelligently carry out these set forms he must, of course, have had instruction in the details of the processes involved. However, if he has taken the mathematical courses and has secured the benefits already enumerated,—the knowledge that he has a mathematical mind, the training in clarity of thinking and the broadening effect of the culture,—he can forget most of the things which he put down on his final examination paper and still do good mathematical work in industrial life.

Do not misunderstand me as saying that because industrial mathematical work is carried out by means of set forms and tables and engineers' pocket books that the training in the elementary processes of the text-books is of no benefit. On the contrary, the set processes alluded to most assuredly cannot be successfully carried out unless the young man has had training in the elements at their foundation.

The fifth and least important direction in which secondary school mathematics is of benefit is in memory training. By this I mean training of that section of the brain which enables us to say things by heart. This is quite a different thing from training the mind in clear thinking previously alluded to. Mind training is training of reasoning faculties and memory training is not. Often this matter of memory thinking is the only benefit which a course in mathematics gives and it is counted as sufficient reason for pursuing the course. I do not agree to this opinion at all. I consider memory training as so much less important than the previous matters that a mathematics course is prostituted if it is principally directed to this feature.

It is obvious that if a young man is to obtain benefit from his mathematical training in the five directions just enumerated, he must have a special aptitude for mathematical work. The discussion of this matter of aptitude which we will now take up I consider the most important point in the whole paper.

Pupils in secondary school mathematical courses can be divided into three classes. First there are those who have so-called mathematical minds and who are at the head of their classes either with little effort or because they enjoy making an effort. The second class pass their subject and to all intents and purposes are doing good work. However, they really know nothing about mathematics as it truly is. They only learn by heart certan rules and can do certain problems only if couched in certain language with which they are familiar. This class includes many girls. Such pupils obtain little or no benefit from high school mathematics as usually taught. The next class of pupils obviously have no talent for mathematical work and do little or nothing. They may be very good pupils and make earnest effort but can accomplish nothing. The last two classes are equally incapable of obtaining mathematical knowledge which will be of benefit to them in industrial life, and must be grouped together as non-mathematicians.

Ability to do mathematical work is just as inherent as is ability to sing in tune or to draw. It is unfortunate that this is not more commonly recognized and instruction for the types of pupils above mentioned arranged accordingly. Take the case of instruction in music or art. No one attempts to make a player out of a person who cannot carry a tune or an artist out of a person who cannot draw. One who can know a tune or can draw is given instruction in all of the technical processes. One who cannot know a tune or cannot draw can nevertheless be given considerable non-technical instruction in music or art. He can be given information regarding the character of the technical processes being used by those who have inherent ability. He can learn of the history of the subject and of the great masters, of the influence on civilization and other similar subjects. No time is wasted in trying to make him hold a pencil or handle a bow. In the case of mathematical work the fundamental aptitude or inaptitude is just as marked and yet we throw all kinds of students together and never seek to separate those who can actually use mathematical processes, and those who cannot use them but could be given instruction in their general nature.

A large majority of the subjects taught in elementary algebra are simply tools of the mathematician's trade. Pupils of the second class above mentioned can apparently learn how to go through the processes involved. However, they can never really use them as tools of the trade. The master violinist may teach his pupil exactly how to hold the bow in his hands and how to hold his fingers in position for the various notes. The pupil may be able to do exactly as he should in these particulars. Yet nothing is accomplished in the way of violin performance unless the pupil has the "divine fire." A boy may be taught to sharpen a wood chisel and to hold it in the proper way in his hands but he can never be a wood-worker unless he originally possessed an aptitude. So a teacher of algebra may make a pupil multiply trinominals, clear of brackets, and perform the other mechanical operations of algebra, but unless the pupil has a mathematical mind he is simply parroting and can never make any real use of the knowledge he has gained.

In case of grammar school pupils it is necessary that they go through a certain number of things even if they are not fitted for them and do them in parrot way. In the case of a student who is preparing for college a similar situation probably arises although I am by no means certain of this. Still for the sake of argument I am willing to admit that a college student must learn a certain number of things for which he is not adapted in order to intelligently pursue certain other subjects upon which they bear indirectly and for which he may be better adapted. In the case of a student whose school work ceases with the secondary school no such situation arises and there is no reason whatever for giving him instruction in a subject for which he is not fitted. It is no disgrace to a student if he is not fitted for mathematical work. He may be a master in some other direction. Each one of us has a certain direction in which we are best adapted to shine in life and on which we should concentrate most of our energies. The things in this field we can really do. Other things we may learn about in order to broaden our outlook but we should not try to learn to do them, as we never can.

Suppose one had a mixed class of babies and puppies and set about teaching them to walk on two legs. He could teach them all something or other if he put enough time on it. Exactly the same situation exists in any of our classes in algebra. There are some fitted to learn algebra just as well as a baby is fitted to walk on two legs and others fitted just as well as a puppy is to walk on two legs. Furthermore, trying to teach the latter algebra has just as much utility as trying to teach the puppy to walk. The puppy may be a perfectly good puppy and may be able to run and hunt and do all of the other things which a good dog should do but he is just as much out of his sphere when walking on two legs as the baby is at catching a squirrel.

I urge, then, that very early in the secondary school course separation be made of those pupils who can and who cannot pursue mathematical subjects with profit and that thereafter the mathematicians be not held back by the non-mathematicians and the non-mathematicians be given other subjects in which they can spend their time more profitably. I would like to emphasize a suggestion which I believe has been made often before, that courses be arranged to tell pupils about mathematics who are not fitted to use mathematics. Such courses would include some of the history of mathematics, than which no chapter in history is more interesting or more important. The history of geometry; the story of Euclid, whose original arrangement has needed no essential change to this day; the study of the Hindoo and Egyptian geometers, the part that geometry plays in the marvelous mechanical advances of today and a host of similar subjects will give pupils with non-mathematical minds much more culture and memory training than the present method of trying to make them go through elementary processes in a halting way. There is a great need for a text-book on the "Story of Mathematics" suited for secondary schools and when the masterpiece in this direction is written there will be a great change in our present methods of instruction of non-mathematicians.

We have now enumerated five directions in which a student secures benefit from mathematical training in industrial life: 1st, vocational directing; 2d, mind training; 3d, culture; 4th, detailed mathematical knowledge; 5th, memory training; and have divided the pupils into two classes, the mathematicians and non-mathematicians.

We will conclude by discussing how mathematical instruction should be directed so as to best secure to the two classes of pupils the greatest possible benefit in the five directions.

In the matter of vocational directing the teacher has little to do other than to lead the pupil along to the full extent of his mathematical powers if he is fitted for mathematical work and to lead him to select other subjects if he is not fitted.

In the matter of the second point, mind training, only those who have mathematical minds can secure any benefit no matter how the mathematical teaching is conducted. I believe that no gain in clarity of thought but rather the reverse can come if a non-mathematician pursues a subject in which he is more or less muddled all the time. The non-mathematicians are therefore wholly barred from training of reasoning powers by mathematical work and there is no use to attempt it.

In the matter of culture the mathematical pupils of course receive a considerable amount from their mathematical work. The presumption is that they enjoy it and are fully awake to all of the processes involved. Therefore, they cannot but help being broadened and being given the advantages which accrue by heart of the mathematical rules even though to the non-mathmaticians never really participate in mathematical subjects as they are usually taught and therefore receive no benefit in this direction. If, however, some instruction such as I have suggested in the way of history or story of mathematics can be arranged, non-mathematical pupils could receive considerable culture.

In the matter of the fifth point, memory training, both mathematicians and non-mathematicians receive benefit. Learning by heart of the mathematical rules even though to the non-mathematician they are meaningless trains the memory. However, such memory training for the non-mathematician could just as well be secured in other directions where there will be other benefits besides.

In the matter of the fourth benefit, detailed mathematical knowledge, the non-mathematical pupil can be given as much and the mathematician as little of this as you please, and the latter will soon outstrip the former in any practical use of mathematics. Even in the most routine types of mathematical work in industrial life a non-mathematician, no matter how much he has learned by heart, can never do well, so that training in this direction can never do him any real good.

There remains only to discuss how the subject matter of secondary school mathematics can be arranged to best give detailed mathematical knowledge to the pupils with mathematical minds. As I have already mentioned, most mathematical processes in industrial life are carried on according to certain set forms which are usually learned at the time. No one is called upon to originate any such methods until he has obtained considerable facility in the work by holding subordinate positions for many years. Therefore the mathematical training should be directed as much as possible towards fundamentals and as little as possible towards details.

To take up specific subjects let us begin with algebra. We of course must include the fundamental processes. However, the manipulative details such as factoring, least common multiple, long division, solution of numerical equations and similar subjects can be omitted entirely or greatly abbreviated. On the contrary the solution of practical problems should be dwelt upon to a large extent. By these, I mean problems which involve setting down of a given verbal description in mathematical form and obtaining an algebraic equation whose solution must be found. I can express my whole idea by stating that I consider as the most important part of this matter the correct setting down of the verbal description in a mathematical form and the least important part the algebraic manipulation of the resulting equation.

In the case of trigonometry the practical problem side should also be emphasized. I would greatly abbreviate reduction of trigonometric puzzles to simpler forms and would only require memorizing of a very few fundamental formulæ. I would wholly omit solution of oblique triangles. As given in most of the text-books this is purely a mathematical recreation. When an actual oblique triangle is to be solved as in civil engineering, navigation or astronomy by the actual workers in these fields the usual text-book methods are not used and certain set forms and tables are used instead.

In the case of geometry I believe in covering a comparatively small amount of ground thoroughly. In this as in all cases effort must be made to give a thorough training in fundamental principles, remembering that this will leave a permanent impression on the mind when detailed demonstrations which have been learned have been long forgotten.

Of solid geometry I would give only that portion relating to mensuration and wholly omit all of the remainder.

Spherical trigonometry is occasionally made a high school subject. This I do not believe should be done as high school pupils are not of sufficient maturity to grasp it properly. Furthermore, it has little or no use in industrial life.

As has been obvious from the paper, the most important point to my mind in this matter of the relation of secondary school mathematics to industrial life is the omission of detailed mathematical instruction for non-mathematicians and concentration of it for mathematicians. I hope some reform will occur in this respect.